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ESTIMATION OF THE SPECIFIC HEAT AND THERMAL CONDUCTIVITY OF FOODS ONLY BY THEIR CLASSES OF SUBSTANCES CONTENTS (WATER, PROTEINS, FATS, CARBOHYDRATES, FIBERS AND ASH)

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Abstract

It is presented an easy way to calculate specific heat and thermal conductivity for foods using the percentile contents by classes of substances (water, proteins, fats, carbohydrates, fibers and ash) using the versatile MathCad program, dedicated for mathematical calculus and graphical presentations.

***Keywords**: foods, estimation, specific heat, thermal conductivity, MathCAD*

Introduction

Specific heat and thermal conductivity are the most important foods' technological characteristics used to solve the heat balances and heat transfer problems. Mathcad combines the live document interface of a spreadsheet with the WYSIWYG (what you see is what you get) interface of a word processor. In addition, Mathcad's computational abilities range from adding up a column of numbers, text, graphics to evaluating integrals and derivatives, solving systems of equations, and more (MathCAD, 2001).

Results and Discussions

For fat-free fruits and vegetables, purees, and concentrates of plants origin, Siebel (1918) observed that the specific heat varies with moistures contents and that the specific heat can be determined as the weighted mean of the specific water and the specific heat of the solids.

For a fat free plant material with a fraction of water – M, the specific heat above freezing point is 4186.8 J/(kg·K), and for nonfat solids is 837.36 J/(kg·K). So in SI, Ashare (1965) proposed the weighted average specific heat for a unit mass of material above the freezing point as:

$$C_{avg} = 3349.2 \cdot M + 837.36 \quad \mathrm{J/(kg \cdot K)} \tag{1}$$

Below the freezing point the Ashare's relationship is:

$$C_{avg} = 1256 \cdot M + 837.36 \quad \mathrm{J/(kg \cdot K)} \tag{2}$$

Another more general relationships are the next for above the freezing point:

$$C'_{avg} = 1674.72 \cdot F + 837.36 \cdot SNF + 4186.8 \cdot M \quad \mathrm{J/(kg \cdot K)} \tag{4}$$

and below the freezing point:

$$C'_{avg} = 1674.72 \cdot F + 837.36 \cdot SNF + 2093.4 \cdot M \quad \mathrm{J/(kg \cdot K)} \tag{5}$$

where F is representing fat, mass fraction nonfat – SNF, and mass fraction moisture – M.

A more appropriate way to estimate the specific heat of solids and liquids are the correlations obtained by Choi and Okos (1987).

The specific heats, in J/(kg·K), as a function of t (in Celsius degrees) for various components of foods are expressed as follows, for:

Proteins : $C_{pp} = 2008.2 + 1208.9 \cdot 10^{-3} \cdot t - 1312.9 \cdot 10^{-6} \cdot t^2$ (6)

Fats : $C_{pf} = 1984.2 + 1473.3 \cdot 10^{-3} \cdot t - 4800.8 \cdot 10^{-6} \cdot t^2$ (7)

Carbohydrates : $C_{pc} = 1548.8 + 1962.5 \cdot 10^{-3} \cdot t - 5939.9 \cdot 10^{-6} \cdot t^2$ (8)

Fibers : $C_{pfi} = 1845.9 + 1930.6 \cdot 10^{-3} \cdot t - 4650.9 \cdot 10^{-6} \cdot t^2$ (9)

Ash : $C_{pa} = 1092.6 + 1889.6 \cdot 10^{-3} \cdot t - 3681.7 \cdot 10^{-6} \cdot t^2$ (10)

Water above freezing point:

$$C_{wf} = 4176.2 - 9.0862 \cdot 10^{-5} \cdot t + 5473.1 \cdot 10^{-6} \cdot t^2 \tag{11}$$

Then the specific heat of the mixture above the freezing point is:

$$C_{avg} = P \cdot C_{pp} + F \cdot C_{pf} + C \cdot C_{ps} + Fi \cdot C_{pfi} + A \cdot C_{pa} + M \cdot C_{waf} \tag{12}$$

For enthalpy changes calculations, Choi and Okos' equations for specific heat must be expressed as an average over the range of temperatures under consideration. The mean specific heat – C_m, over a temperature range t_1 to t_2, where $\left(t_2 - t_1\right) = \delta$, $\left(t_2^2 - t_1^2\right) = \delta^2$ and $\left(t_2^3 - t_1^3\right) = \delta^3$ is:

$$C_m = \frac{1}{\delta} \int_{t_1}^{t_2} C_p dt \tag{13}$$

So for various components over the temperature range – δ, the equations for the mean specific heats become (Toledo, 1994):

$$C_{mpp} = (1/\delta) \cdot [2008{,}2 \cdot \delta + 0{,}6045 \cdot \delta^2 - 437{,}6 \cdot 10^{-6} \cdot \delta^3] \tag{14}$$

$$C_{mpf} = (1/\delta)\cdot[1984{,}2\cdot\delta + 0{,}7367\cdot\delta^2 - 1600\cdot10^{-6}\cdot\delta^3] \quad (15)$$

$$C_{mpc} = (1/\delta)\cdot[1548{,}8\cdot\delta + 0{,}9812\cdot\delta^2 - 1980\cdot10^{-6}\cdot\delta^3] \quad (16)$$

$$C_{mpfi} = (1/\delta)\cdot[1845{,}9\cdot\delta + 0{,}9653\cdot\delta^2 - 1500\cdot10^{-6}\cdot\delta^3] \quad (16)$$

$$C_{mpa} = (1/\delta)\cdot[1092{,}6\cdot\delta + 0{,}9448\cdot\delta^2 - 1227\cdot10^{-6}\cdot\delta^3] \quad (17)$$

$$C_{mwaf} = (1/\delta)\cdot[4176{,}2\cdot\delta - 4{,}543\cdot10^{-5}\cdot\delta^2 + 1824\cdot10^{-6}\cdot\delta^3] \quad (18)$$

$$C_{mavg} = P\cdot C_{mpp} + F\cdot C_{mpf} + C\cdot C_{mps} + Fi\cdot C_{mpfi} + A\cdot C_{mpa} + M\cdot C_{mwaf} \quad (19)$$

For the estimation of thermal conductivity of food products taking into account the effect of variations in the composition of a material, Choi and Okos (1987) reported the following procedure.

The thermal conductivity – λ of a product is estimated as a sum of products between the conductivity of pure components – λ_i and the volume fraction of each component - x_{vi}.

$$\lambda = \sum \lambda_i \cdot x_{vi} \text{ , W/(m·K)} \quad (20)$$

$$\lambda_{\mathrm{w}} = 0.57109 + 1.7625\cdot10^{-3}\cdot t - 6.7306\cdot10^{-6}\cdot t^2 \text{, W/(m·K)} \quad (21)$$

$$\lambda_{\mathrm{ic}} = 2.2196 - 6.2489\cdot10^{-3}\cdot t + 1.0154\cdot10^{-4}\cdot t^2 \text{, W/(m·K)} \quad (22)$$

$$\lambda_{\mathrm{p}} = 0.1788 + 1.1958\cdot10^{-3}\cdot t - 2.7178\cdot10^{-6}\cdot t^2 \text{, W/(m·K)} \quad (23)$$

$$\lambda_{\mathrm{f}} = 0.1807 - 2.7604\cdot10^{-3}\cdot t - 1.7749\cdot10^{-7}\cdot t^2 \text{, W/(m·K)} \quad (24)$$

$$\lambda_{\mathrm{c}} = 0.2014 + 1.3874\cdot10^{-3}\cdot t - 4.3312\cdot10^{-6}\cdot t^2 \text{, W/(m·K)} \quad (25)$$

$$\lambda_{\mathrm{fi}} = 0.18331 + 1.2497\cdot10^{-3}\cdot t - 3.1683\cdot10^{-6}\cdot t^2 \text{, W/(m·K)} \quad (26)$$

$$\lambda_a = 0.3296 + 1.401\cdot10^{-3}\cdot t - 2.9069\cdot10^{-6}\cdot t^2 \text{, W/(m·K)} \quad (27)$$

The volume fraction – x_{vi}, of each component is determined from the mass fraction – x_i, the individual density – ρ_I, and the composite density - ρ, as follows:

$$x_{vi} = \frac{x_i \cdot \rho}{\rho_i} \quad (28) \qquad \rho = \frac{1}{\sum (x_i / \rho_i)} \quad (29)$$

The individual densities, in kg/m^3, for water - ρ_w, ice - ρ_{ic}, protein - ρ_p, fat - ρ_f, carbohydrate - ρ_c, fiber - ρ_{fi}, and ash - ρ_a, are:

$$\rho_{\mathrm{w}} = 997.18 + 3.1439\cdot10^{-3}\cdot t - 3.7574\cdot10^{-3}\cdot t^2 \quad (30)$$

$$\rho_{ic} = 916.89 - 0{,}13071\cdot t \quad (31)$$

$$\rho_p = 1329.9 - 0.51814\cdot t \quad (32)$$

$$\rho_f = 925.59 - 0.41757 \cdot t \tag{33}$$

$$\rho_c = 1599.1 - 0.31046 \cdot t \tag{34}$$

$$\rho_{fi} = 1311.5 - 0.36589 \cdot t \tag{35}$$

$$\rho_a = 2423.8 - 0.28063 \cdot t \tag{36}$$

Using equations (20)–(36) thermal conductivity (figures 1 and 2) and density (figure 3) for a lean pork composition: 71.7% water, 19.0% protein, 7.8% fat, and 1.5% ash using MathCad utilities was estimated.

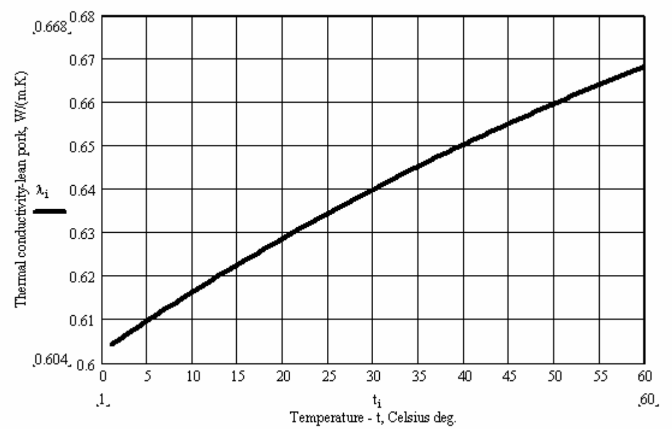

Fig. 1. Thermal conductivity estimation for lean pork (temperature 1 – 60°C)

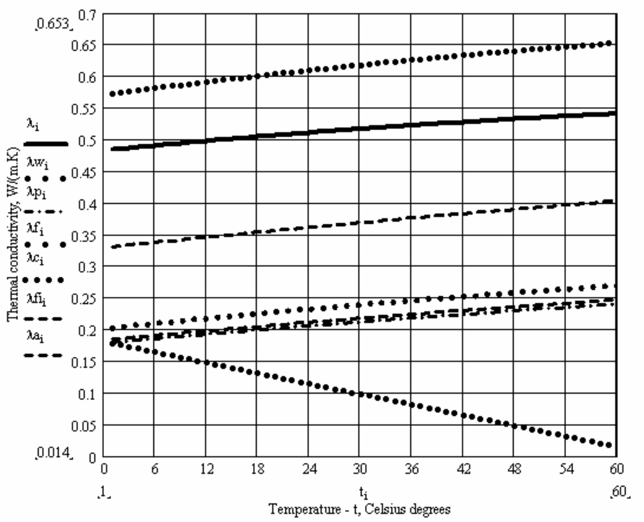

Fig. 2. Thermal conductivity variation of the lean pork's components

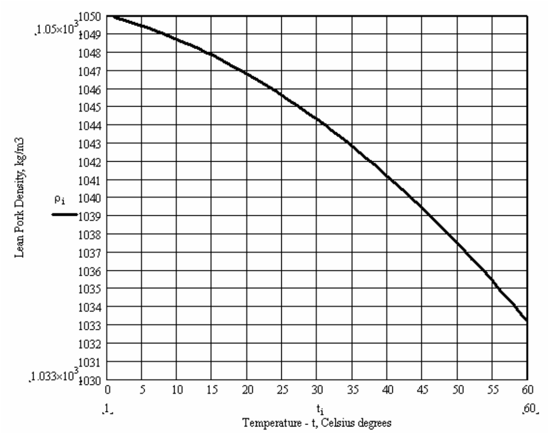

Fig. 3. Lean pork's density as a temperature function

For a sort of milk with the composition: water – 87.3%; proteins – 3.7%; fats – 3.8%; carbohydrates – 4.6% and ash – 0.6%, thermal conductivity (figure 4), density (figure 5) and thermal conductivity for milk's components were estimated.

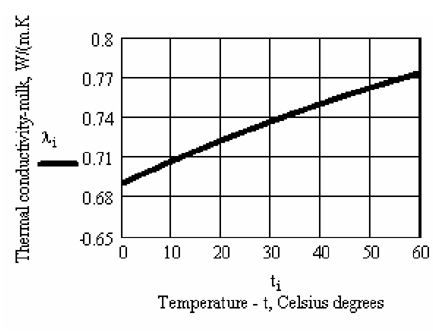

Fig. 4. Thermal conductivity estimation for milk

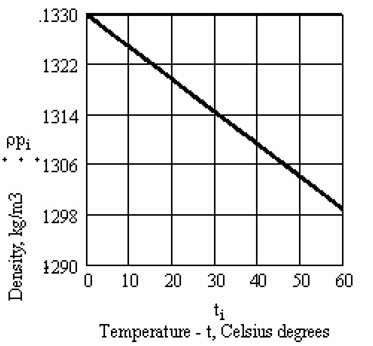

Fig. 5. Milk density as a temperature function

Values for C_p calculated using Choi and Okos' (1987) correlations, are generally higher than those calculated using Siebel's equations at high moisture contents ($M > 0,70$). Choi and Okos' correlations are more accurate at low moisture contents and for a wider range of product composition (Macovei, 2000; Onița, 2004).

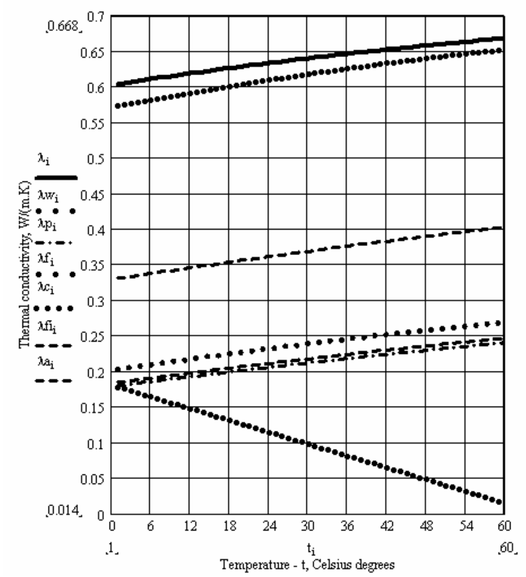

Fig. 6. Thermal conductivity estimation for milk's components

Conclusions

It was proved that MathCad is a powerful tool of research in the field of foods research, especially in the cases where little information there are or quite nothing about thermal properties of a food material, but only the contents by classes of substances: water, ice, proteins, fats, carbohydrates, fibers and minerals (Toledo, 1994).

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